Introducing Physics Students to Electric Circuit Components

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Abstract

Most courses on linear electric circuits state models and solutions using directly observable circuit variables. This is the best way to introduce the theory for circuit design. We recommend using some features available in some textbooks to improve teaching of electric circuit theory to all students, and some features to better explain the electromagnetic phenomena in terms of electromagnetic fields.

Contents

1 Introduction

Most college students of physics and engineering learn about the application-rich theory of lumped-parameter linear dynamic systems. These courses focus on stating models and solutions with observable system variables: voltage drop and current through each a component, and their time derivatives. This is the best approach to prepare students for circuit design activities.

This work suggests focusing on two kinds of information that is found in some textbooks: first, presenting formulas for combining components in series or in parallel in ways that improve students' intuitive grasp of the material; secondly, for students interested in physics, developing key results in terms of magnetic fields and fluxes.

2 Resistors

A discrete circuit component has constant resistance R if it dissipates electrical energy according to:

(1) $V = R\, I$ power dissipated: DC: $P = I\, V = I^2 R$ AC: $\int_0^{\text{period}} I^2 R\, dt\, /\text{period}$

For resistors R1 and R2 in series, the combined resistance is R = (R1+R2), which is easily understood by most students.

For two resistors R1 and R2 in parallel, V = V1 = V2 and I = I1 + I2. So

(2) $I = I1 + I2 = (1/R1 + 1/R2)\, V$ that is, $I = G\, V$ where conductance $G = G1 + G2$

We believe the formula for combining parallel resistors makes more intuitive sense when presented with conductance.

3 Capacitors

A discrete circuit component has constant capacitance C if applying a voltage V across a component causes charge Q to be stored in the component, according to:

(3) Static relation: $Q = C\, V$ Dynamic relation: $I = C\, dV/dt$

3.1 Energy stored by a capacitor

PE can be expressed in terms of circuit variables as the energy needed to push Q "uphill" in the electrical potential.

(4) $$PE = \int Q\, (dV/dt)\, dt = \int C\, V\, dV = ½\, C\, V^2$$

Consider a parallel plate capacitor, where A = area of a plate, d = distance between plates, and ε = dielectric constant of the medium. Substitute V = – E d and C = ε A / d into equation (4):

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(5) $PE = ½ ∫ \mathbf{\underline{E}}•\mathbf{\underline{D}}\ dvol$

The energy stored by the capacitor is stored as the energy of the electric field inside the capacitor [1 p 795; 2 p 487].

3.2 Capacitors in series

For two capacitors in parallel, V = V1 = V2 and Q = Q1 + Q2. So Q = C V where C = C1 +C2, which is easily understood by most students.

For two capacitors in series, V = V1 + V2 and I = I1 = I2. So dV1/dt = 1/C1 I1 and dV2/dt = 1/C2 I2, and

(6) $dV/dt = dV1/dt + dV2/dt = 1/C1\ I1+1/C2\ I2 = 1/C\ I$ where $1/C = 1/C1 +1/C2$

We believe this result makes more sense to students when expressed with elastance E = 1/C. [3 /Elastance]

(7) static: $V = E\ Q$ dynamic: $dV/dt = E\ I$ where $E = E1 + E2$

A mechanical analogy might help introduce elastance to students: (a) V ~ pressure pushing fluid into a tank; (b) Q ~ quantity of fluid already in a tank, and I ~ rate of fluid flow into the tank; (c) E ~ elastic stiffness of the tank that opposes the flow in proportion to the amount of fluid in the tank.

(8) $V = E\ Q$ mechanical analog: pressure = elastic stiffness (fluid in tank)

3.3 Current through a capacitor

A capacitor has no electrical continuity between its nodes. Current flows in the sense that dQ/dt at the two nodes has the same magnitude and opposite signs. We recommend that physics students learn this meaning of current flow.

4 Inductors

A discrete circuit component has inductance if applying dI/dt across a component causes a voltage difference – ΔV across the component. This process implies that applying – ΔV across the component causes the component to store energy.

4.1 Inductance and potential energy expressed with circuit variables

Inductors store charge and energy without dissipation. Linear capacitance can be defined with circuit variables as

(9) constitutive relation: $– V = L\ dI/dt$ definition of linear capacitance: $L := – V/(dI/dt)$

This definition of inductance yields an expression for potential energy (PE) stored by an inductor using circuit variables.

(10) $PE = ∫ (– V)\ I\ dt = ∫ (L\ dI/dt)\ I\ dt = ∫ L\ I\ dI$ so: $PE = ½ L I^2$

These expression for inductance and potential energy are the most useful form for circuit design.

4.2 Inductance in terms of the magnetic field

By far the simplest useful example of an inductor is a solenoid: a tightly-wound helical coil of wire. Consider a solenoid of radius R, length Len >> R, n turns per unit length, with magnetic permeability μ outside and inside that carries current I.

By the Ampere-Maxwell law, the current generates a magnetic field **B** inside and outside the solenoid.

In order to define inductance in terms of the magnetic field, define the magnetic flux. For any surface in space, and any magnetic field **B**, the magnetic flux is define as:

(11) $Φ_B := ∫_{Area} \mathbf{\underline{B}} • \mathbf{\underline{n}}\ dArea$

Using cylindrical coordinates (r, φ, z), we focus on the surface z = 0 that intersects the solenoid at mid-length. Decompose $Φ_B$ into parts inside and outside the solenoid.

(12) $Φ_B := Φ_{B,in} + Φ_{B,out} = 2 π ∫_0^R \mathbf{\underline{B}} • \mathbf{\underline{n}}\ dr + 2 π ∫_R^∞ \mathbf{\underline{B}} • \mathbf{\underline{n}}\ dr$)

References [1] and [2] establish that in the limit Len/R → ∞,

(13) $\mathbf{\underline{B}}_{z,in} = μ\ n\ I$ $\mathbf{\underline{B}}_{z,out} = 0$

(14) $Φ_{B,in} = μ\ n\ I\ π\ R^2$ $Φ_{B,out} = 0$

By the integral version of Faraday’s law (∂Area = boundary of the disk at z = 0 inside the solenoid),

(15) $\partial_t \Phi_{B,in} + \int \underline{\mathbf{E}} \cdot \underline{\mathbf{t}}\, d(\partial Area) = 0$ so: $\partial_t \Phi_{B,in} = -\, emf = V$

Substituting the right equation in (15) into the right equation in (9), express inductance in terms of circuit variables.

(16) $L = \partial_t \Phi_B / (d\mathbf{I}/dt) = \mu\, n\, \pi\, R^2$

In linear circuits, L is constant, so from equation (16) we can infer that

(17) $L = \Phi_B / \mathbf{I}$

4.3 Potential energy of an inductor expressed in terms of magnetic field

Consider the solenoid. Solve the left equation in (13) for **I**.

(18) $\mathbf{I} = B_{in} / (\mu\, n\, Len)$

To express PE in term of **B**, substitute equation (18) into right-hand equation (10) to get:

(19) $PE = ½ (B_{in}^2/\mu)\,(Area\ Len)$ so: $PE = ½ \int \underline{\mathbf{B}} \bullet \underline{\mathbf{H}}\, dvol$

All energy stored by an inductor is stored in the magnetic field in space near the inductor.
Equate expressions for PE in (10) and (19) and solve for L to get

(20) $L = \int \underline{\mathbf{B}} \bullet \underline{\mathbf{H}}\, dvol / \mathbf{I}^2 = PE / (½\, \mathbf{I}^2)$

Since equations (10) and (19) are independent of the solenoid model, equation (20) is as well.

4.4 Inductors in parallel and in series

For two inductors in series, **I** = **I**1 = **I**2 and V = V1 + V2. So V = L d**I**/dt where L =L1 + L2, which is easily understood by most students.

For inductors in parallel, V = V1 = V2 and **I** = **I**1 + **I**2. So inductance of the combined components is described by:

(21) $d\mathbf{I}/dt = (1/L1 + 1/L2)\, V = (1/L)\, V$ where $L = 1/L1 + 1/L2 = L1\, L2/(L1 + L2)$

We believe this result makes more sense to students when expressed with reluctance R = 1/L [3 /Magnetic_reluctance].

(22) $d\mathbf{I}/dt = R\, V$ where $R = R1 + R2$

A mechanical analogy might help introduce students to magnetic reluctance: (a) **I** ~ conserved quantity in a tank; (b) R ~ conductance of a pipe into the tank; (c) V ~ pressure pushing fluid into the tank. This analogy may not be the best one, because **I** is normally the flow of a conserved quantity.

(23) $d\mathbf{I}/dt = R\, V$ mechanical analog: flow rate = conductance pressure

Inductors in parallel or in series are not treated in the textbooks we examined.

4.5 Mutual inductance and magnetic fields outside inductors

We believe the definition of mutual inductance in terms of fields is simpler and more physical than the definition in terms of circuit variables. Mutual inductance is due to B fields outside nearby inductors. [1 p990; 2 p608]. To the extent that **B1** and **B2** of two inductors overlap in space and are parallel, the potential energy of the combined inductors is not the sum of the energies of the separate inductors.

(24) $PE_{total} = ½ \int \underline{\mathbf{B}} \bullet \underline{\mathbf{B}}/\mu\, dvol = ½ \int \underline{\mathbf{B}}\mathbf{1} \bullet \underline{\mathbf{B}}\mathbf{1}/\mu\, dvol + ½ \int \underline{\mathbf{B}}\mathbf{2} \bullet \underline{\mathbf{B}}\mathbf{2}/\mu\, dvol + \int \underline{\mathbf{B}}\mathbf{1} \bullet \underline{\mathbf{B}}\mathbf{2}/\mu\, dvol$

(25) $PE_{total} = PE1 + PE2 + \int \underline{\mathbf{B}}\mathbf{1} \bullet \underline{\mathbf{B}}\mathbf{2}/\mu\, dvol$

4.6 Conditions of validity of solutions

Two conditions apply to solutions for $B_{\square,out}$ for a long wire of radius R and for $B_{z,out}$ for a long solenoid of radius R:
(a) the solutions are valid only for R < r << Len; (b) the solution is not exact for r > R.

Table 1: Treatment by two references of conditions on solution B_{out}

Condition	Long straight wire $B_{\square,out} = \mu\, \mathbf{I} / (2\, \pi\, r)$	Long solenoid $B_{z,out} = \mu\, \mathbf{I} / (2\, \pi\, r)$

Solution is valid only for R < r << Len	Ref [1]: condition is mentioned Ref [2]: condition is not mentioned	Ref [1]: condition is not mentioned Ref [2]: solution not presented
Solution is approximate for R < r << Len	Ref [1]: condition is not mentioned Ref [2]: condition is not mentioned	Ref [1]: condition is not mentioned Ref [2]: solution not presented

These conditions should accompany the description of the solution. If an inquisitive student tries to compute pe = potential energy per unit length for either configuration without these conditions, he/she will get

(26) pe = integrate(constant/r^2 (2 π r dr), R, big) = 2 π constant (ln(big) – ln(R)) → ∞.

If the two conditions above are taken into account and B_{out} ~ constant/r^k with k > 1 for large r, then pe is finite.

If we take limits Len →∞ and r → ∞, the order of the limits matters; more generally, the relative rates of variables approaching infinity. Here is a simple illustration of the problem of the order of limits.

(27) $\text{limit}_{y\to 0}(\text{limit}_{x\to 0}(x^y)) = 0$ but $\text{limit}_{x\to 0}(\text{limit}_{y\to 0}(x^y)) = 1$.

Another example is integrate(integrate(f(x, y) dx) dy) and integrate(integrate(f(x, y) dy) dx), which are equal only under conditions specified by Fubini's theorem.

5 Conclusion

We recommend that students with a strong interest in physics should learn material about circuit components that goes beyond that which is of most interest to engineers: (a) Derivation of constitutive equations and stored energy in terms of electromagnetic fields; (b) A more physical approach to deriving component parameters for parallel and series configurations of some circuit components; (c) a statement of mutual inductance based on magnetic fields; and (d) more accurate statements of the limitations of certain "solutions" for the magnetic field surrounding solenoids.

6 References

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